

PLASMON DECAY: FROM QED TO QCD

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ABSTRACT

Upon using the same theoretical framework, I describe two interesting decay processes: the electromagnetic plasmon decay into neutrinos, which can be the dominant cooling mechanism for red giants and white dwarfs, and the gluonic plasmon decay into quarks, which can be measured in ultra-relativistic heavy-ion collisions.

1. Introduction

Plasmons are collective excitations of bosonic type. Phonons, photons or gluons build up plasmons in a thermal environment. One important feature of the plasmon mode is certainly its mass, usually proportional to gT , where g is the coupling constant of the theory and T the temperature. Being massive, this mode can decay into lighter particles. This phenomenon cannot happen in vacuum, photons and gluons being strictly massless.

Here, I will describe two different processes: the QED plasmon decay into neutrino-antineutrino pairs, which is of crucial relevance to stellar cooling, and the QCD plasmon decay into quark-antiquark pairs, which could be measured at future accelerators if a quark-gluon plasma is formed in heavy-ion collisions.

2. Plasmon decay in QED

The plasmon decay process is one of the dominant cooling mechanisms for stars composed of a degenerate core. It is particularly relevant for the stellar evolution of white dwarfs and red giants. In these two systems, the core is composed of degenerate matter, with density $\rho \simeq 10^6 \text{g/cm}^3$ and temperature in the range $T = 10^7$ - 10^8K . All electronic levels below the Fermi sphere are occupied. A typical value of the electron momentum is $p_F = 400 \text{keV}$.

The plasmon decay process was first considered a long time ago, by Adams, Ruderman and Woo¹, who were followed by many others². The subject has recently been revived when it was realized that the plasmon dispersion relations that were used did not incorporate the relativistic effects³. The latest works that compensate for these effects are due to Itoh et al.⁴, Braaten and Segel⁵, and Haft, Raffelt and Weiss⁶.

Here, I present a slightly different method for calculating the plasmon decay process⁷. What is first needed is the effective neutrino coupling with the electromagnetic field. Within the Standard Model, one-loop thermal corrections bring in an effective charge for a neutrino in a medium⁸:

$$\Gamma^\mu = ie_\nu \gamma^\mu L, \quad (1)$$

where $L = \frac{1}{2}(1 - \gamma_5)$ is the standard left-handed projector and the effective charge e_ν is given by

$$e_\nu = \frac{2\sqrt{2}}{e} G_F c_V \omega_\beta^2, \quad (2)$$

with $c_V = \frac{1}{2} + 2\sin^2 \theta_W$. The neutrino effective charge is matter-dependent through the parameter

$$\omega_\beta^2 = \frac{e^2}{2\pi^2} p_F E_F \left[1 - \frac{1 - v_F^2}{2v_F} \ln \frac{1 + v_F}{1 - v_F} \right], \quad (3)$$

calculated for the case of a degenerate electron gas, where $v_F = p_F/E_F$ is the Fermi velocity.

Because of their weak interactions, the neutrinos emitted from the star are not thermalized and escape freely from the system. Unlike photons, they can drain the energy from the core of the star and cool it down more quickly. Using the cutting rules of Kobes and Semenoff⁹, the neutrino (antineutrino) production rate due to transverse photon decay is given by

$$R_T = \frac{dN_\nu}{d^4x} = e_\nu^2 \int \frac{d^4Q}{(2\pi)^4} \int \frac{d^4K}{(2\pi)^4} 2\pi\delta(K^2) 2\pi\delta((Q - K)^2) (n_B(\omega) + \theta(-q^0)) \times 2\pi\delta(Q^2 - \text{Re}\Pi_T(Q)) \text{Tr} [K\gamma_\mu (\not{K} + \not{Q})\gamma_\nu] \mathcal{P}_T^{\mu\nu}. \quad (4)$$

In this equation, K and Q are the neutrino and photon four-momenta, respectively, and n_B is the Bose-Einstein distribution function. The transverse photon projection operator is $\mathcal{P}_T^{ij} = -\delta^{ij} + q^i q^j / q^2$, with all other components zero¹⁰.

After performing the integrations, one arrives at the final analytic expression

$$R_T = \frac{e_\nu^2}{24\pi^3} \int_0^\infty \frac{q^2 dq}{\omega} Z_T(Q) n_B(\omega) Q^2, \quad (5)$$

where $Z_T(Q)$ is the Jacobian from the $\delta(Q^2 - \text{Re}\Pi_T(Q))$ integration. It has a complicated form⁵ (numerically, its value is never far from 1). It must be understood that in Eq. (5) ω and q are related by a dispersion relation:

$$\omega^2 - q^2 = \frac{3}{2} \omega_0^2 \left[\frac{\omega^2}{v_F^2 q^2} + \frac{1}{2v_F} \frac{\omega}{q} \left(1 - \frac{\omega^2}{v_F^2 q^2} \right) \ln \frac{\omega + v_F q}{\omega - v_F q} \right], \quad (6)$$

where ω_0 is the plasmon frequency given by

$$\omega_0^2 = \frac{e^2}{3\pi^2} \frac{p_F^3}{E_F}. \quad (7)$$

Similar relations can be derived for the longitudinal case. These dispersion relations were first derived by Jancovici¹¹, but had a much more complicated form. It was realized later that simplified expressions could be used in a much wider regime^{5,12}. Note also that for the two stellar systems of physical interest, namely the white dwarfs and the red giants, one can completely neglect the temperature effects in the plasmon dispersion relations.

The expression for the longitudinal emissivity is exactly the same as in Eq. (5), apart from an overall factor 1/2. However, one should not forget that the dispersion relations quite differ.

To be honest, Eq. (5) is only valid when the plasmon is sufficiently close to the light-cone. The neutrino effective charge is in fact a charge radius¹⁻⁷:

$$e_\nu = \frac{2\sqrt{2}}{e} G_F c_V Q^2, \quad (8)$$

and one has $Q^2 = \omega_\beta^2$ when $\omega, q \gg \omega_0$, as it should.

Finally, the energy loss rate due to $\nu\bar{\nu}$ emission is simply obtained by multiplying Eq. (5) by the photon energy ω under the integral.

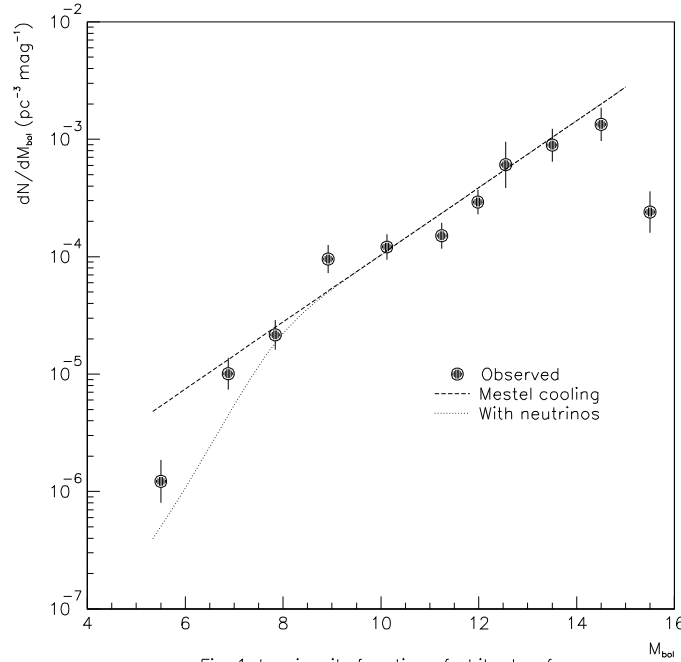


Fig. 1 Luminosity function of white dwarfs

Using the non-relativistic dispersion relations, one easily recovers the old standard result¹ (taking $\sin^2 \theta_W = 1/2$)

$$\epsilon_T^{NR} = \frac{G_F^2}{48\pi^4 \alpha} \omega_0^6 \int_{\omega_0}^{\infty} d\omega \frac{\omega \sqrt{\omega^2 - \omega_0^2}}{e^{\beta\omega} - 1}. \quad (9)$$

The effect of the additional cooling mechanism due to plasmon decay is shown in Fig. 1. At the early stages of the white dwarf evolution, the neutrino luminosity can be five times more important than the photon luminosity. It can be shown that the plasmon decay dominates over other competing neutrino-emitting processes⁶.

3. Plasmon decay in QCD

The possibility that ultra-relativistic nuclear collisions may create a quark-gluon plasma is extremely interesting. Particularly relevant to this problem are the different time scales which are involved in these collisions: the thermalization time,

the chemical equilibration time, etc.... Recent work¹³ has shown that, while the hot matter should thermalize in about 0.3 fm/c, it may be largely a gluon plasma (GP), with few quarks. Chemical equilibration is expected to take much longer than thermal equilibration, if it occurs at all.

Let me consider the idealized situation of a GP or QGP in thermal equilibrium. If such a plasma does exist, the massless gluons evolve into quasi-particles with effective masses of order gT . Being massive, these quasi-gluons decay into $q\bar{q}$ pairs. This situation is then very similar to the one studied previously.

The plasma frequency (or plasmon mass) in a QCD plasma is given by¹⁰

$$\omega_0^2 = \left(N + \frac{N_f}{2}\right) \frac{g^2 T^2}{9}, \quad (9)$$

for $SU(N)$ gauge theory at temperature T , where g is the strong coupling constant and N_f is the number of massless fermion flavours. Thermal effects in QGP and GP are thus identical, except that $N_f = 0$ for GP.

The gluon decay is an obvious mechanism for the production of $q\bar{q}$ pairs. It has been overlooked in the past literature¹⁴. The starting equation for the rate is exactly the same as in Eq. (4), except for the coupling. However, unlike the photon, the gluon has an anomalously large damping rate¹⁵:

$$\gamma_g = 3\alpha_S T \ln \frac{\omega_0}{m_{mag}} + O(\alpha_S), \quad (10)$$

where $m_{mag} = O(g^2 T)$ is the magnetic mass. Therefore, instead of having a $\delta(Q^2 - \text{Re}\Pi(Q))$ for the gluon propagator, one has a Lorentzian with width γ_g .

When this effect is taken into account, one finds for the transverse gluon decay (and for massless quarks)¹⁶:

$$R_{g \rightarrow q\bar{q}}^T = \frac{2g^2\gamma_g}{9\pi^4} \int_0^\infty d\omega \, \omega^2 \, n_B(\omega) \left\{ \ln \frac{64\omega^4}{9\omega_0^4 + 16\gamma_g^2\omega^2} + \frac{3\omega_0^2}{2\gamma_g\omega} \left(\arctan \frac{3\omega_0^2}{4\gamma_g\omega} + \arctan \frac{2\omega}{\gamma_g} \right) - 4 \right\}, \quad (11)$$

where terms of higher order than g^4 have been dropped. The limit $\gamma_g \rightarrow 0$ reproduces the result using a gluon propagator without a finite damping rate (see Eq. (5)).

Together with the longitudinal contribution, the final result is

$$R_{g \rightarrow q\bar{q}} = \frac{2\zeta(3)}{\pi^3} \alpha_S^2 \left(\ln \frac{1}{\alpha_S} \right)^2 T^4 + \mathcal{O} \left(\alpha_S^2 \ln \frac{1}{\alpha_S} T^4 \right). \quad (12)$$

Notice that using a bare gluon propagator, $\delta(Q^2)$, would lead to a vanishing result. With just the hard thermal mass, $\delta(Q^2 - 3\omega_0^2/2)$, the rate is of order $g^4 T^4$, with no logarithmic dependence. Taking into account the anomalously large damping rate γ_g shifts the gluon on-shellness $Q^2 \sim \omega_0^2$ by a logarithmic correction. The additional logarithm has a kinematic origin and comes from the pole of the gluon propagator that is almost on shell.

The gluon decay process is the leading contribution in the perturbative expansion, i.e. when $g \rightarrow 0$. Diagrams that were calculated in previous works, as $gg \rightarrow q\bar{q}$ and $q\bar{q} \rightarrow q\bar{q}$, are subleading compared to this process (although just by a log).

Even though, numerical results show that chemical equilibration will proceed very slowly, with $\tau_q > 10 T^{-1}$, even at $g = 3$, for the very light quarks¹⁶.

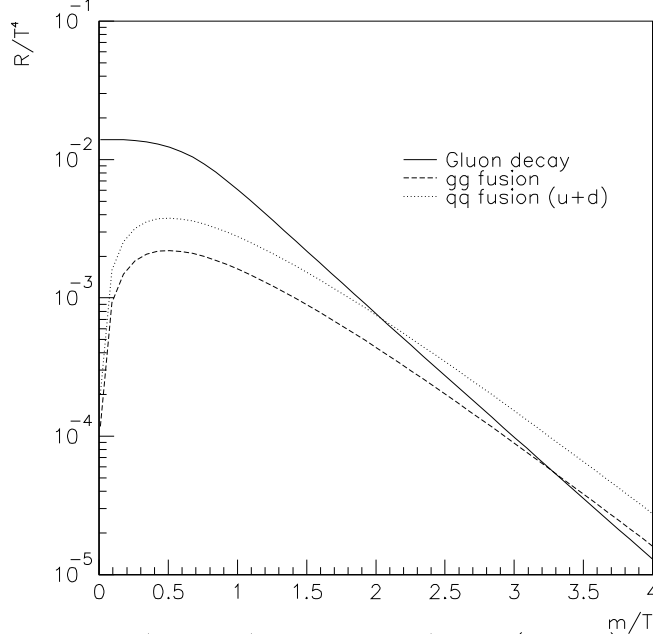


Fig. 2 Massive quark production rate ($\alpha_s=0.32$)^{m/T}

The picture is a bit changed when massive quarks are considered. When $M \gg T$, one gets for the rates¹⁷

$$\begin{aligned} R_{g \rightarrow Q\bar{Q}} &= \frac{2\alpha_S}{\pi^2} \gamma_g T^3 e^{-2M/T} \\ R_{gg \rightarrow Q\bar{Q}} &= \frac{7\alpha_S^2}{6\pi^2} MT^3 e^{-2M/T} \\ R_{q\bar{q} \rightarrow Q\bar{Q}} &= \frac{\alpha_S^2}{\pi^2} MT^3 e^{-2M/T}. \end{aligned} \tag{13}$$

These rates are plotted in Fig. 2. The gluon decay dominates for light ($M < 2T$) quarks. Certainly, one should be able to measure the gluon damping rate, and therefore the magnetic mass, by looking at the strange or charm quark production in heavy-ion collisions.

4. Conclusion

In the two examples I have given, Thermal Field Theory has proved its power and its usefulness. In the first case, the analytic expressions are much simpler to manipulate than those obtained by using Kinetic Theory. This complexity may have been the reason why relativistic effects were overlooked for 30 years. The second example is an even better one, of the confusion that can arise by using the Kinetic

Theory. It is very encouraging to see that the wonderful structure of hot QCD can be tested in relativistic heavy-ion collisions. This is especially true for the gluon damping rate, to which so much theoretical work has been devoted.

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